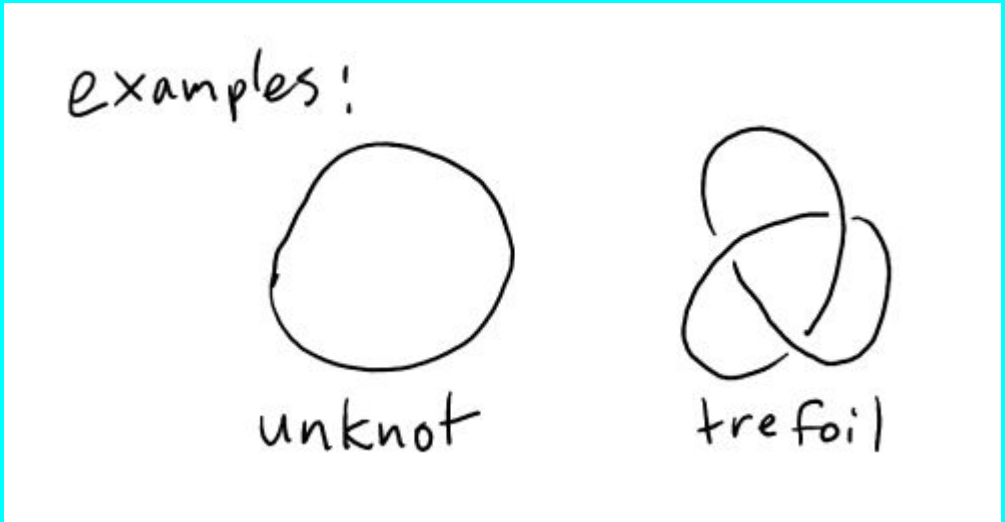
knot theory

(illustrations added)

what is a knot?

- a knot is a loop in 3-dimensional space



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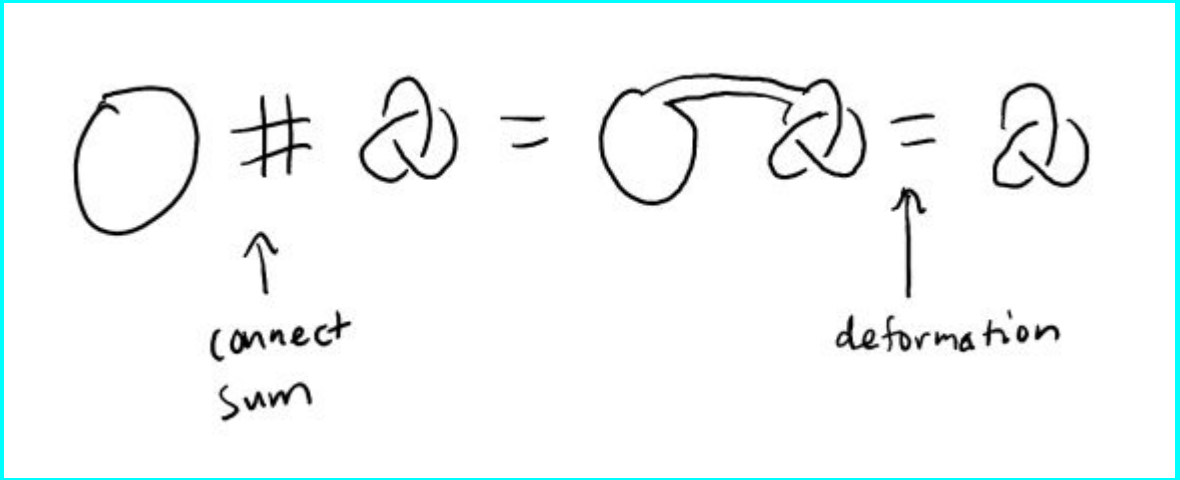
- represent knots by diagrams with crossings
- goal: classify the knots!

the “connect” sum

- an operation, but with knots!

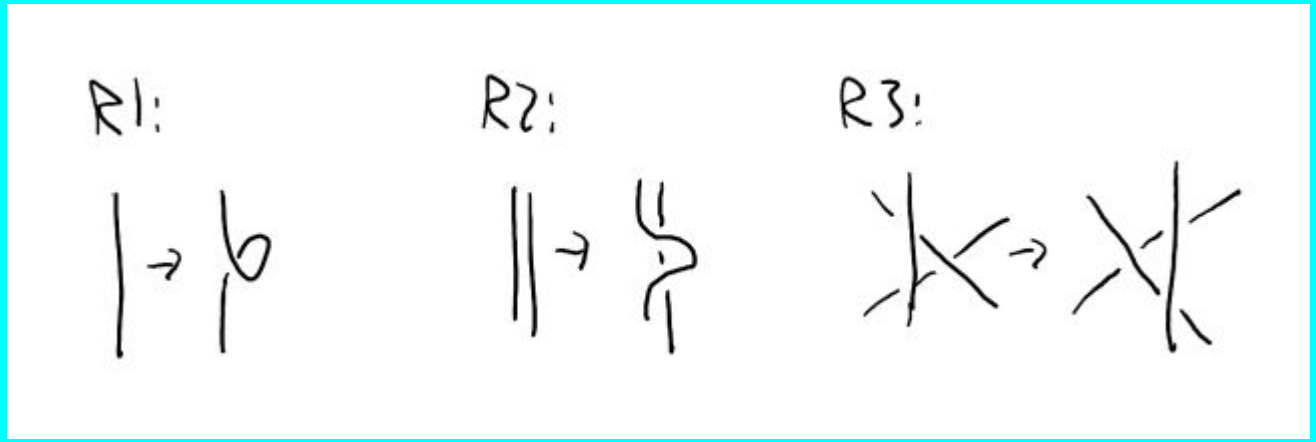


the “connect” sum

- an operation, but with knots!
- satisfies the commutative property

how to tell (prove) that two knots are the same

- theorem: two diagrams represent the same knot if and only if their diagrams are related by a sequence of Reidemeister moves (and their inverses)



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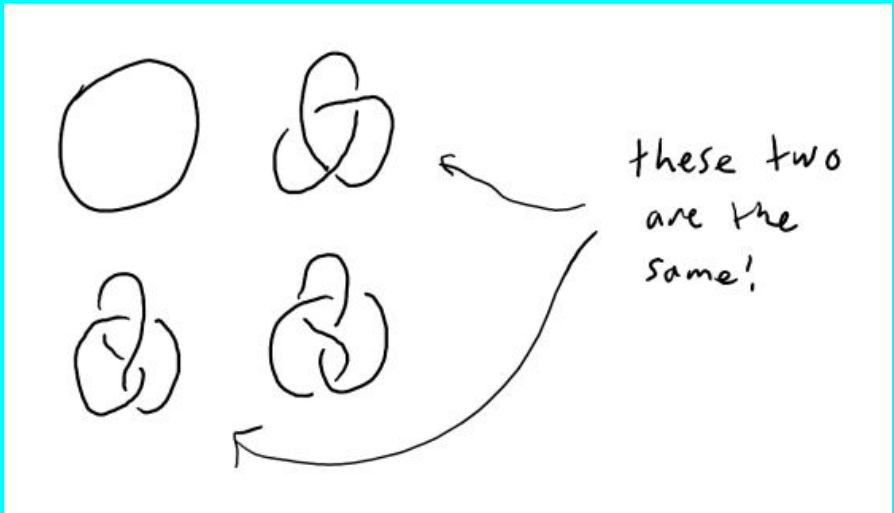

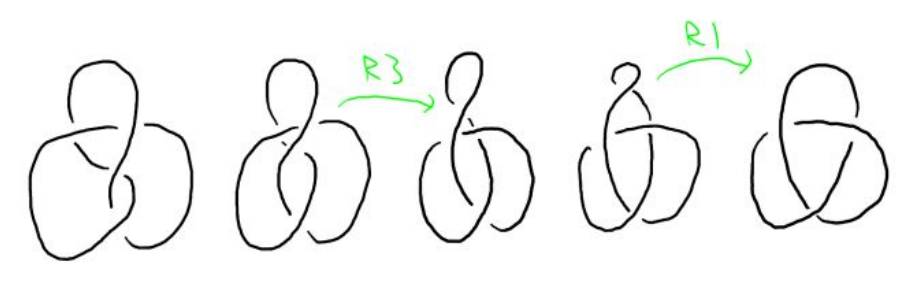

how to tell (prove) that two knots are the same

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- example
- can we use Reidemeister moves to tell if two knots are different?

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    - nope

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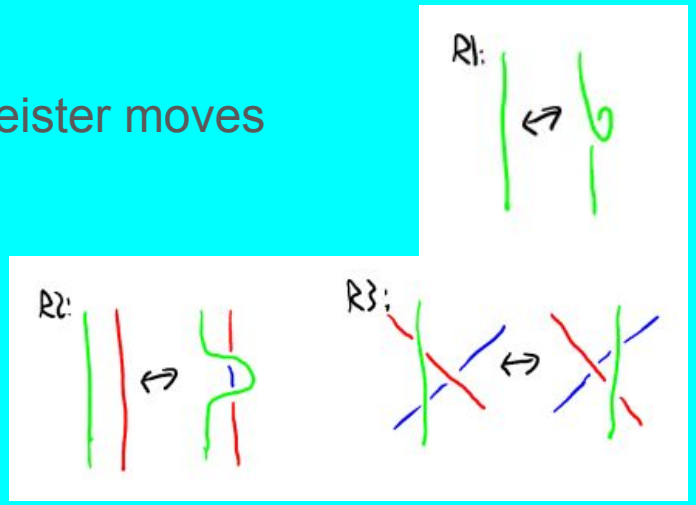
how to tell (prove) that two knots are different

- idea: tell knot diagrams apart by their properties
- a knot diagram is tricolorable if you can draw it with three colors such that each intersection has either one color or three colors
- the unknot is not tricolorable, but the trefoil is!
- theorem: tricolorability is invariant under Reidemeister moves

in other words, if we do Reidemeister moves on a tricolorable knot, the result will still be tricolorable, and the same for non-tricolorable knots. proof: if a knot has a valid coloring on it, we can use the following colorings after a Reidemeister move:



if  and  were equal, then there would be a sequence of Reidemeister moves between them, and then either both or neither must be tricolorable. but  is not tricolorable and  is. therefore, the two knots are <u>not</u> the same.

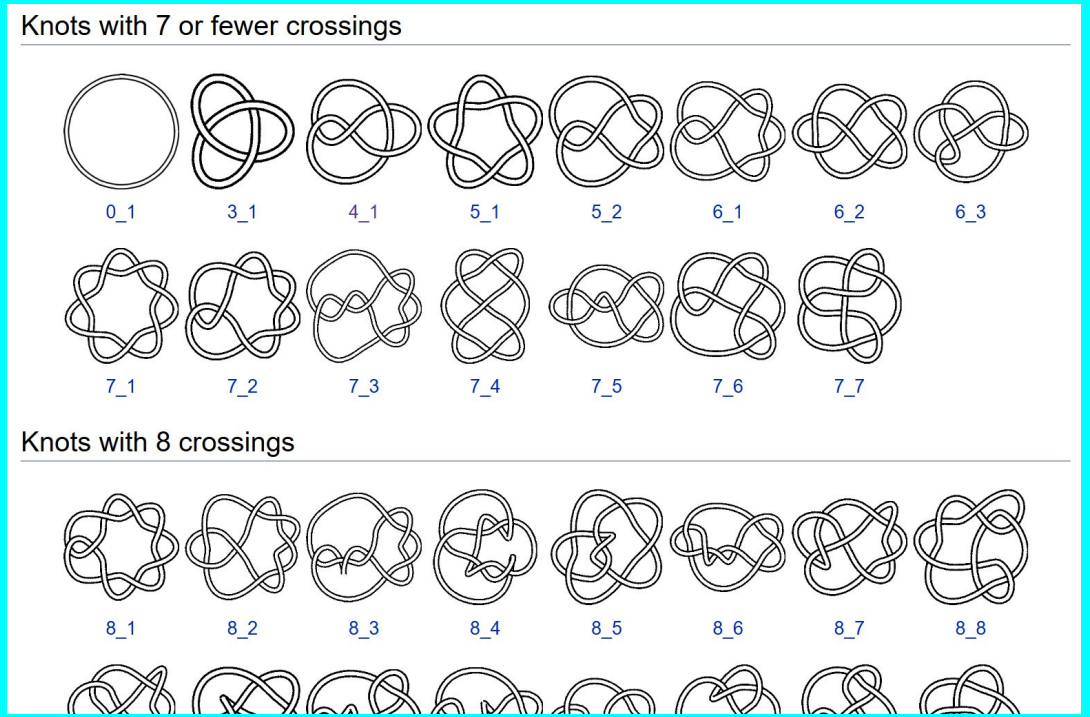
not tricolorable! can't use three colors

tricolorable!

not tricolorable. must use 4 colors

knot “size” numbers

- crossing number



knot “size” numbers

- crossing number
- unknotting number

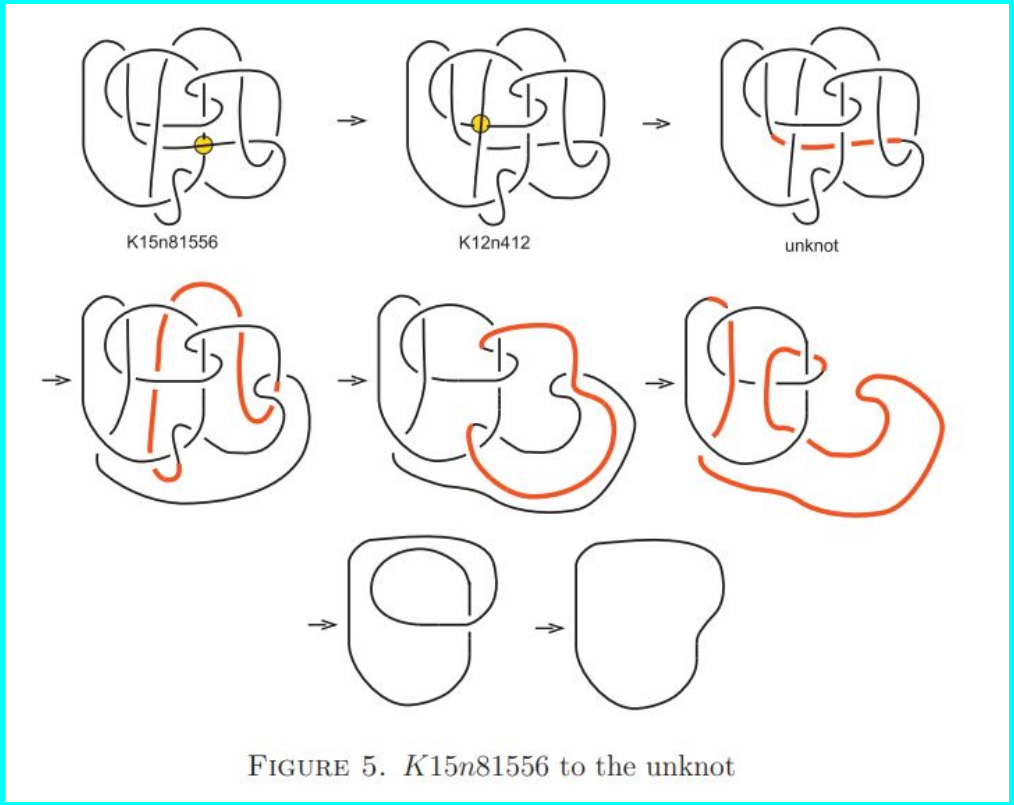


FIGURE 5. $K15n81556$ to the unknot

knot “size” numbers

- crossing number
- unknotting number
- tile number

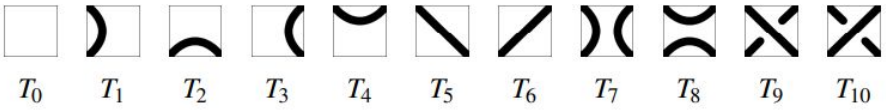
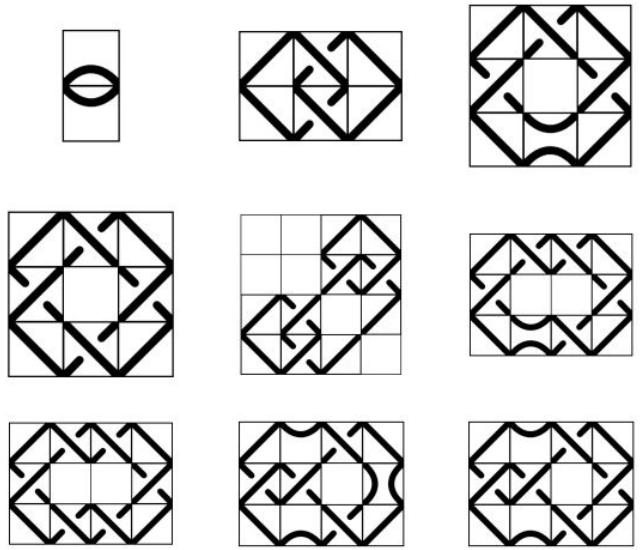


Figure 2: The corner tiles.

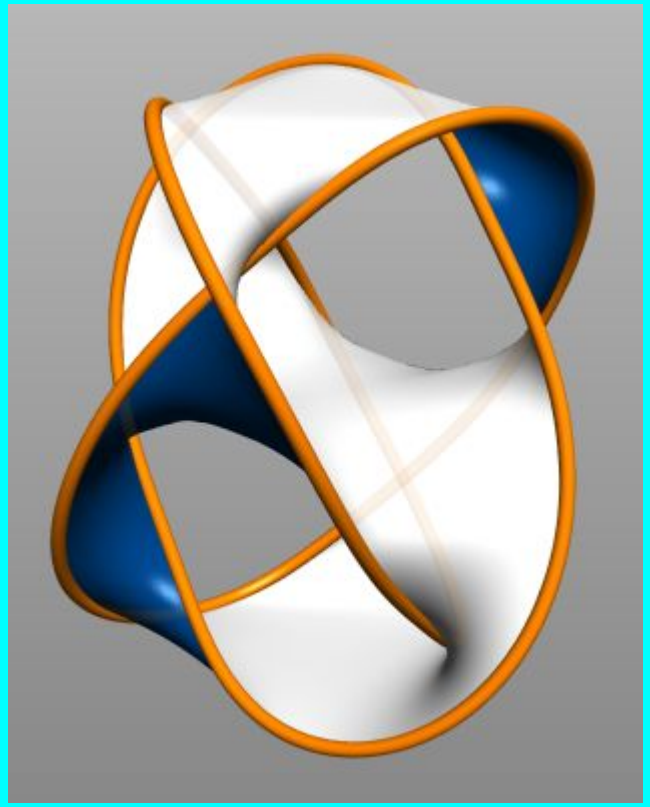


knot polynomials

- Alexander polynomial
- Jones polynomial
  - multiplicative!

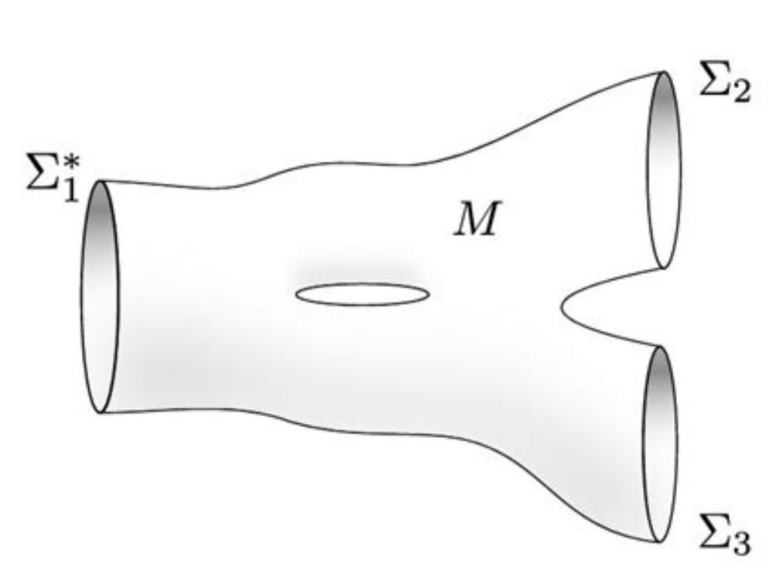
knot genus

- seifert genus
- slice genus



knot concordance

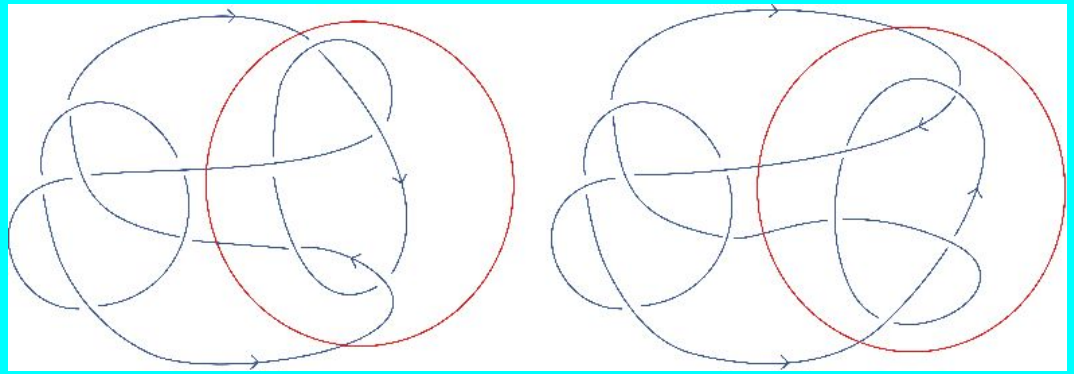
roughly, two knots are in the same concordance class if they can be connected by a surface:



the Conway knot

a knot is slice if it is in the same concordance class as the unknot

it was open for 50 years whether the Conway knot was slice, until Lisa Piccirillo, a grad student at UT Austin, found that it is not!



generalizations

- wild knots
- two dimensional knots
- knot “embedding space”

questions?